**ENEE 660 Fall 2010 Homework 2 (due in class Thursday, September 16, 2010)**

1. Show that the determinant of a matrix is equal to the product of its eigenvalues. Show that the trace of a matrix is equal to the sum of its eigenvalues.

2. A vector space $V$ over a field $F$ can be given the structure of an *inner product space* if we define an inner product on it. When the field $F$ is the set $\mathbb{R}$ of real numbers, we call this a real inner product space. A real inner product $\langle,\rangle : V \times V \to \mathbb{R}$ is defined by the following axioms:

$\langle v, w\rangle = \langle w, v\rangle \quad (symmetry)$

$\langle v, au + bw\rangle = a\langle v, u\rangle + b\langle v, w\rangle \quad (bilinearity)$

$\langle u, u\rangle \geq 0, and\ \langle u, u\rangle = 0, if\ and\ only\ if \quad u = 0 \quad (positive\ \ definiteness)$

Here $u, v, w$ denote vectors, and $a, b$ denote scalars $= \mathbb{R}$.

Show that the vector space $V = \mathbb{R}^n$ with the so called standard inner product

$$\langle u, v\rangle = \sum_i^n u_i v_i$$

satisfies the axioms above. Suppose that $A$ is a $n \times n$ real matrix satisfying the property that it preserves inner products, i.e.

$$\langle Au, Au\rangle = \langle u, u\rangle, \quad for\ all\ u \in V.$$

Then, show that, all the eigenvalues of $A$ are of unit magnitude. Are they real? Always? Sometimes? Never?

3. Read sections 5.1 and 5.2 in the textbook. Do exercise 5.2, part (a).

4. Read sections 6.1 and 6.2 in the textbook. For the matrices in exercise 6.2, compute the matrix exponential $e^{tA}$.

5. Express the transition matrix of

$$A = \begin{pmatrix} 0 & a \\ -a & 0 \end{pmatrix}$$

in closed form by summing its series representation by hand.