ENEE 661

Homework Set 3 (Part 2) due 02/21/2013

4. Let $s \mapsto \gamma(s)$, $a < s < b$, $\gamma(s) \in \mathbb{R}^3$ be a unit speed curve. Let $s \mapsto T(s)$ denote the associated unit tangent vector, varying with s.

(a) Under what conditions is the curve $s \mapsto T(s)$ a regular curve?

(b) Suppose κ and τ are the curvature and torsion functions of the curve γ. Suppose $\kappa > 0$. Let $\bar{s}$ denote the unit speed parametrization of T, and let $\bar{\kappa}$ denote the curvature function of T. Show that

$$\bar{\kappa} = \sqrt{1 + (\tau/\kappa)^2}$$

5. Let $I\dot{\Omega} = I\Omega \times \Omega + U$ denote the Euler equation for a rigid body (e.g. spinning spacecraft) subject to an external torque U with components u_i $i = 1, 2, 3$. The first term in the Euler equation is a drift term governing spin when $u_i \equiv 0 \ \forall i = 1, 2, 3$. The parameter

$$\mathbb{I} = \mathrm{diag}(I_1, I_2, I_3).$$

The vector Ω = body angular velocity.

$\mathbb{I}$ is the diagonal matrix of principal moments of inertia of the rigid body.

Is it possible to steer the rigid body from one spin state $\Omega(0)$ to a final spin state $\Omega(1)$ using only one of the three controls, say u_1, turned on and the other two turned off?

Here $I_1 \geq I_2 \geq I_3 > 0$