

ENEE 664 OPTIMAL CONTROL Spring 2012 Homework 1 (due back 02/02/2012)

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1. Write a complete proof of the (Fredholm Alternative) Theorem:

Let  $V$  and  $W$  be two finite dimensional vector spaces with well-defined inner products on them. Let  $A: V \rightarrow W$  be a linear mapping. Then,

$Ax = b$  has a solution,

if and only if

for every  $p$  in  $\text{Ker}(A^*)$ ,  $\langle p, b \rangle = 0$ .

(Here  $A^*$  denotes the adjoint of  $A$  and  $\text{Ker}(A^*) = \text{null-space of } A^*$  and  $\langle, \rangle$  denotes the inner product on  $W$ . You may refer to Appendix A of Professor Tits' lecture notes - see course website.)

From this show that  $\text{Range}(A) = \text{Range}(AA^*)$

2. Read (Lecture Notes 1 ENEE 664, and as background on linear systems, Lecture Notes 1 and 2 of ENEE 660 System Theory - see link in course webpage)