ENEE 664 OPTIMAL CONTROL Spring 2014 Homework 1 (due back 02/05/2014)

1. Write a complete proof of the (Fredholm Alternative) Theorem:

Let V and W be two finite dimensional vector spaces with well-defined inner products on them. Let A: V ---> W be a linear mapping. Then,

$Ax = b$ has a solution,

if and only if

for every p in Ker(A*), $\langle p , b \rangle = 0$.

(Here A* denotes the adjoint of A and Ker(A*) = null-space of A* and < , > denotes the inner product on W. You may refer to Appendix A of Professor Tits' lecture notes - see course website.)

From this show that Range (A) = Range (AA*)

2. Read (Lecture Notes 1 ENEE 664, and as background on linear systems, Lecture Notes 1 and 2 of ENEE 660 System Theory - see link in course webpage)

3. On page 10 of the Lecture Notes 1, compute the efficiency and determine how it changes if a series inductor of value L is included.

4. Suppose the A and B matrices defining a linear control system are time-invariant. Suppose a given initial condition x_0 can be driven to the origin in time T. Can it be driven to the origin in half that time? If so what is the change in the optimal quadratic cost (used in Lecture 1)? You may investigate this second part of the problem through a simple but illuminating example (first).